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The axial skew of flow in curved pipes

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As a uniformly coiled pipe is wound progressively tighter, centrifugal acceleration would be expected to drive the axial flow increasingly towards the outer wall of the pipe bend. Instead, the effect of bend curvature R is found experimentally to become fully expressed at small curvature ratios ($a/R < 0.02$), where a is the pipe radius. No further increase in the axial skew is observed in more tightly coiled pipe sections ($0.02 \leqslant a/R \leqslant 0.20$). In this 'asymptotic' regime where pipe curvature is unimportant, the developing axial skew intensifies as $\approx [(Re-100)\,L/a]^{\frac{1}{4}}$, where $Re = 2\overline{W}a/\nu$ and L is the entrance length. These results suggest that the action of centrifugal force remains balanced by swirl as flow develops in tightly coiled pipes, while in loosely coiled pipes the development of centrifugal effects lags the growth of swirl.

1. Introduction

In a recent review, Berger, Talbot & Yao (1983) have stressed the increased analytic complexity introduced by the presence of stream curvature in conduit flows. Even for steady flow the full Navier–Stokes equations contain several terms involving two independent dimensionless parameters – the curvature ratio a/R and the Dean number $\kappa = (a/R)^{\frac{1}{2}}(2\overline{W}a/\nu)$. As these equations can be simplified considerably by neglecting terms of order $(a/R)^{1}$ or higher, most computational studies have considered only loosely coiled pipes ($a/R \ll 1$) for which the simplified form is valid. For such pipes the reduced equations imply that *all* developing flow patterns should either scale *only* with κ, or else be independent of *both* parameters. While the resulting computations are usually assumed to be valid for $a/R < 0.2$, the existing experimental literature is too qualitative to test this assumption fully. Therefore, a quantitative determination of how developing flow patterns depend on a vanishing curvature ratio would be important in assessing the accuracy of tractable approximation methods. In this report we show that a/R remains an independent parameter in low laminar flows irrespective of any Dean-number effect, so that higher-order terms in a/R must be important for curvature ratios as small as 0.007.

For experimental convenience we chose to examine the developing axial profile produced by steady, uniform, laminar entry flow through uniformly coiled pipes. This flow pattern can be seen in figure 1 to exhibit two characteristic features as it evolves downstream. Near the inlet, stream curvature causes the uniform entry profile to form a potential line vortex, with flow skewed towards the *inner* wall of the pipe bend (Singh 1974). Further downstream, the cumulative action of centrifugal acceleration causes axial flow to shift progressively towards the *outer* wall of the bend. While

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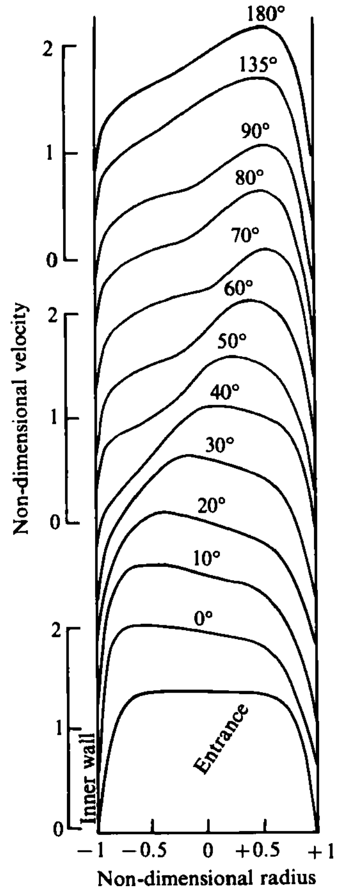

FIGURE 1. The outward shift of developing flow in the symmetry plane of a curved pipe; $a/R = \frac{1}{8}$, $\overline{W} = 22$ cm/s, $Re = 540$. Pipe is bent towards left. Axial skew initially favours the inner wall as the inlet flow profile deforms into a line vortex. With increasing bend angle, the cumulative action of centrifugal acceleration causes skew to shift outwards. Data from Olson (1971), by hot-wire anemometry.

details of these secondary flow patterns may vary from case to case, a consistent picture of how they evolve can be obtained by using integral measures of axial skew. The results shown in figure 2, for example, were constructed from published accounts of axial-velocity profiles in curved pipes simply by evaluating relative flow through each half of the pipe cross-section. This figure makes evident both stages of developing axial skew, and for several values of a/R and κ skew is seen to develop over a lengthscale L/a, where L is the entrance length and a the pipe radius.

Apparently, the pipe radius of curvature R does not influence the outward shift in axial flow over the range of curvature ratios $0.05 \leqslant a/R \leqslant 0.215$ reported in earlier studies by Agrawal, Talbot & Gong (1978) and by Olson & Snyder (1985). As this outcome seemed unlikely on physical grounds, the present investigation used a much wider range of a/R, including cases of vanishing curvature where flow can be expected to be nearly axisymmetric. Van Dyke (1978) has conjectured that in such loosely coiled pipes the character of secondary flow may not depend uniquely on the Dean number κ. We shall show his hypothesis to be essentially correct: at least for developing axial skew, κ is not the sole governing parameter. Either a/R or the Reynolds number $2\overline{W}a/\nu$ must also be considered independently.

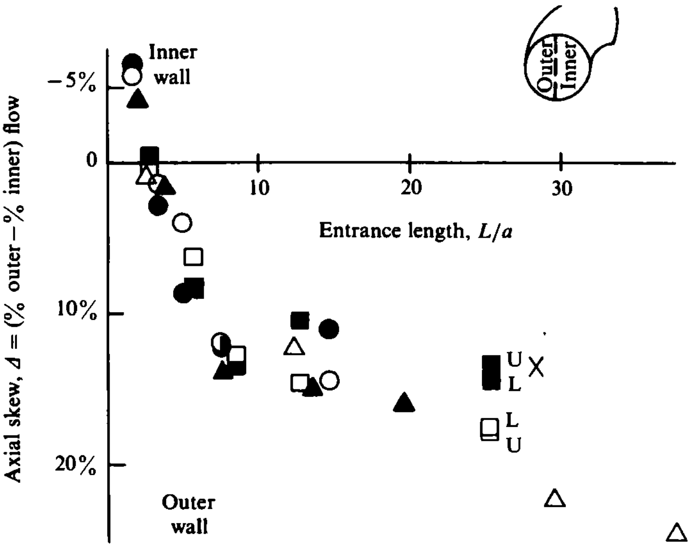

FIGURE 2. The development of axial skew in tightly coiled pipes, as scaled to entrance length L/a. Skew is computed by integrating pointwise measurements of axial velocity. An effect of Re is evident for $L/a \gtrsim 10$, but no effect of a/R is apparent. From Agrawal, Talbot & Gong (1978), using glycerin–water solutions: ▲, $a/R = \frac{1}{7}$, $Re = 485$; △, $\frac{1}{20}$, 2527. From Olson & Snyder (1985), using air: ●, ○, $a/R = \frac{1}{4.66}$, $Re = 300$ and 1080; ■, □, $\frac{1}{8}$, 290 and 1100. Present report: ×, $a/R = \frac{1}{36}$ and $\frac{1}{54}$, $Re = 539$. U = upper half, L = lower half of cross-section.

The limit point of vanishing curvature is also useful in assessing models of entry length L_c for fully developed curved-pipe flow. Yao & Berger (1975) used a modal analysis to obtain the expression $L_c/L_s \simeq 8e_1/\kappa^{\frac{1}{4}}$, where $L_s = 0.125a\,Re$ is the entry length for a straight pipe of equivalent diameter and Reynolds number, while e_1 is a parameter that is thought to depend weakly on a/R ($2 \leqslant e_1 \leqslant 4$). The form of this expression seems doubtful however, since in loosely coiled pipes L_c/L_s would become unbounded at *any* Re as a/R tended to zero. A similar difficulty would arise at low Re, for *any* curvature ratio. As is discussed later, our experimental results suggest a possible resolution of this point.

The intent of this report is not to define entry length, but rather to explore a downstream regime of developing flow, bracketed by two lengths of pipe ($28.2 \leqslant L/a \leqslant 56.5$). For these entrance lengths, flow in straight pipes becomes fully developed for $Re \lesssim 420$. Since these Reynolds numbers are comparable to our experimental cases, the flow in these loosely coiled pipes may also be nearly fully developed. Both entrance lengths extend far beyond the cross-over point $L/a \simeq 2.4$, shown in figure 2, where skew first shifts outwards. Therefore the regime of interest is an asymptotic downstream region where effects of pipe curvature and friction can be expected to govern the pattern of axial flow.

2. Experimental methods

By imagining the pipe cross-section to be bisected along its vertical centreline into 'inner' and 'outer' halves, the axial skew can be defined simply as

$$\Delta = (\%\ \text{flow in outer half}) - (\%\ \text{flow in inner half}). \qquad (1)$$

The quantities on the right-hand side of (1) are measured using the experimental design in figure 3. A length of flexible Tygon tubing 1.91 cm ($\frac{3}{4}$ in.) i.d. is bent to a

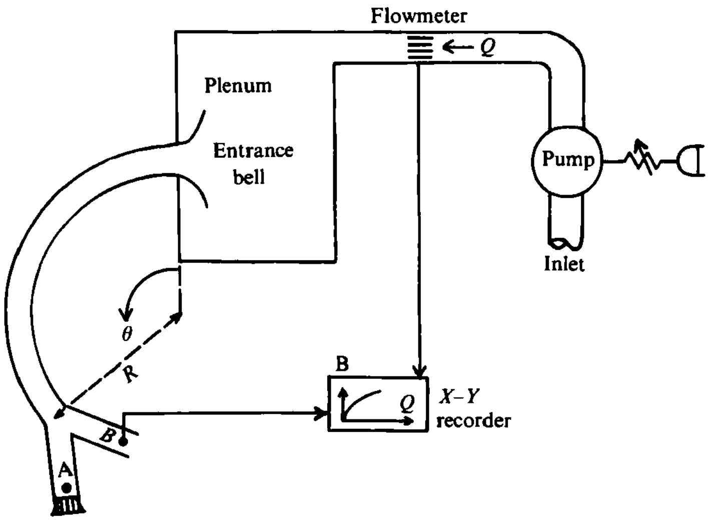

FIGURE 3. Experimental design, showing the upstream flow plenum and entrance bell, the curved pipe, and a downstream bifurcation. The bifurcation was used to split axial flow into an 'inner' and 'outer' flow. Flow in each branch was measured by inserting a hot-wire anemometer into the airstream at A or B. Total flow Q was measured upstream of the plenum. Branch flow and total flow were recorded simultaneously on a digital oscilloscope to obtain flow–flow curves as in figure 4. Here, the anemometer at B is being calibrated to total flow.

desired curvature ratio a/R, mounted on a horizontal plane,† and joined to an entrance bell that provides a nearly uniform inlet-velocity profile. Flow at the pipe exit is split into 'inner' and 'outer' streams by a symmetric 60° bifurcation mounted flush to the tubing. Flow in each stream is measured by inserting a hot-wire anemometer into the centre of the branch cross-section, while total flow is measured by a calibrated laminar-flow meter (linear pneumotachograph) located upstream of the entrance bell. After relating each branch flow to the total inlet flow, (1) is used to compute the axial skew Δ.

Examples of flow–flow tracings generated by simultaneously recording the anemometer and pneumotachograph signals are shown in figure 4 (a, b). It is evident that the anemometer response is nonlinear, but this is of no consequence so long as the pneumotachograph itself is linear with flow, as it is in the low-laminar regime. The anemometer is calibrated *in situ* to the pneumotachograph, by plugging the unmonitored branch. Thus the total flow registered by the pneumotachograph must also pass through the branch containing the anemometer. The resulting calibration curves in figure 4 (a, b), although nonlinear, yield a one-to-one correspondence between the anemometer and pneumotachograph signals. The unmonitored branch is then unplugged to permit flow splitting to occur in the exit bifurcation. The resulting flow–flow curves define relative flow through each branch, and are quantified from the corresponding calibration curve by converting the anemometer signal to branch flow.

The use of a hot-wire anemometer as a volume-flow meter may be novel to some experimenters. The technique is advantageous in that it minimizes exit-flow resistance, which might otherwise distort the axial distribution of flow in the curved pipe

† The tubing remained nearly circular in cross-section even when bent, as it was constrained externally by circular hoops placed at intervals along its length and was relatively thick walled.

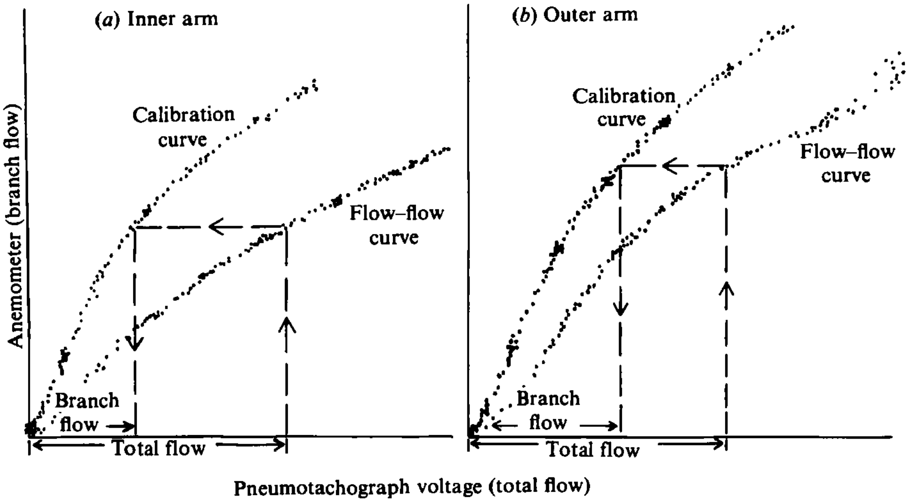

FIGURE 4. Oscilloscope tracings of flow–flow curves for a curved pipe; $L/a = 56.5$, $a/R = \frac{1}{21.6}$. Each calibration curve was generated by directing the entire airstream past the anemometer as flow was slowly increased. After unplugging the unmonitored branch, the procedure was repeated to determine the corresponding flow–flow curve. The dashed lines illustrate how branch flows were obtained. At 75 units of total flow ($Re = 466$), flow–flow curve (*a*) yields 31 units of 'inner' flow and curve (*b*) yields 44 units of 'outer' flow. Hence $\Delta = [(44-31)/75] \times 100\,\% = 17.3\,\%$.

segment. It also permits axial skew to be scanned over a wide range of inlet velocities, simply by using a variable-speed pump to increase flow in a 'quasi-steady' manner. In practice the results are highly reproducible and insensitive to small variations in anemometer position, provided axial-velocity profiles in the downstream segment of each branch remain independent of flow rate. Olson (1971) has shown that flow entering a symmetric bifurcation becomes highly skewed as it passes the flow divider, returning to a nearly symmetric distribution approximately 10 diameters downstream. His tracings, as reproduced in figure 5(*a*), show that at shorter entrance lengths the axial profile changes significantly with Reynolds number. As such shape changes would undermine the correspondence between the calibration curve and the flow–flow curve, the exit branches were extended to provide 9 diameters of flow development between the flow divider and the anemometer.

An effect of flow separation set an upper limit to the useful Reynolds-number range of the technique. The sharply curved junction between the curved pipe and bifurcation produces a strongly separated flow pattern at higher *Re*, causing a flow split that does not reflect the upstream axial profile in the curved pipe. This effect could be detected by an abrupt increase in anemometer noise at $Re \simeq 650$ as shown in figure 5(*b*), and by a failure to conserve total flow when both branch flows were summed. Therefore our reliable measurements were confined to Reynolds numbers of 550 or less.

In addition, it was necessary to quantify the upstream influence of the flow divider on the axial-flow profile in the curved pipe. Humphrey, Taylor & Whitelaw (1977) have described an analogous effect occurring in the entry region of pipe bends, which can be detected 5 diameters upstream of the onset of curvature. Its quantitative significance to our experimental results was evaluated by comparing the effect of different exit conditions on the centreline-velocity profile in the curved pipe. The hot-wire tracings in figure 6 show no detectable upstream effect at the point where

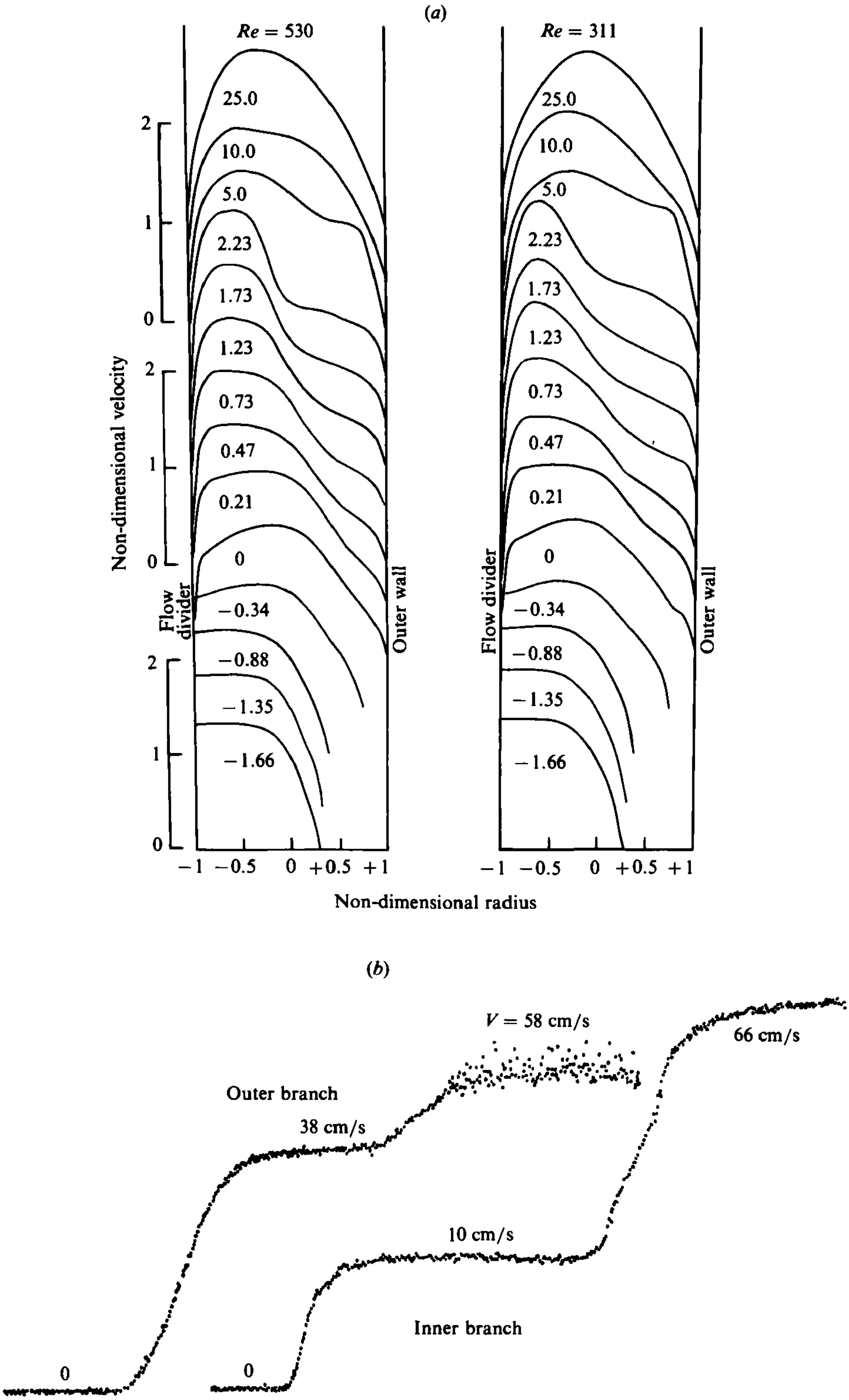

FIGURE 5. For caption see facing page.

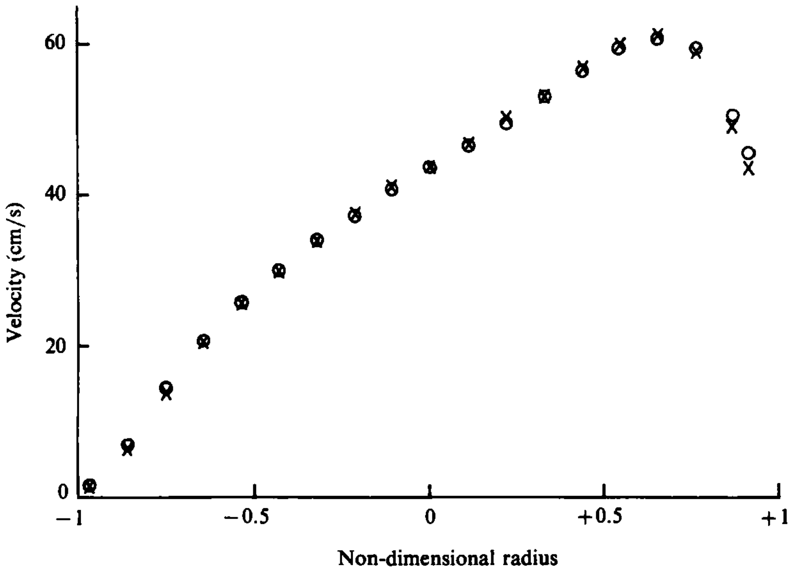

FIGURE 6. Velocity profiles in the symmetry plane of a 90° bend, for two different exit conditions: ×, straight pipe 6 cm long; ○, bifurcation, with the flow divider 1.5 diameters downstream of the hot-wire anemometer. Each point is the average of two measurements, repeatable to ± 0.5 cm/s. $\overline{W} = 42.2$ cm/s, $L/a = 56.5$, $Re = 504$.

the bifurcation begins to diverge, just 1.5 diameters upstream of the flow divider. From these measurements we conclude that the upstream effect of the flow divider is likely to be too localized to affect the measured distribution of flow, at least at low Re.

The experimental technique was further verified by requiring that the sum of branch flows, measured independently, equal total inlet flow to within 2 %; that flow through a straight pipe be measured as axisymmetric; and that skew in tightly coiled pipes reproduce previously reported results (figure 2).

3. Results

Measured values of the axial skew Δ, as defined by (1), are shown in figure 7 (a, b) for entrance lengths L/a of 28.2 and 56.5, respectively. The solid lines in this figure are computed from the empirically derived expression

$$\Delta = 1.38\left[(Re-100)\frac{L}{a}\right]^{\frac{1}{4}}\left\{1-\left[\frac{211}{(1+\theta(1+\frac{1}{2}\pi\theta))\,Re}\right]^{0.142L^2/aR}\right\}$$

$$(L/a \geqslant 10;\ 150 \leqslant Re \leqslant 2600), \qquad (2)$$

FIGURE 5. Limitations of the experimental procedure: (a) Velocity profiles, showing the change in shape on passage through a symmetric bifurcation (from Olson 1971). The flow divider is at $L/D = 0$; positive numbers denote downstream distances in pipe diameters. At $L/D = 5$ the centreline velocity changes disproportionately more than 10 % as Re increases from 311 to 530, while at $L/D = 10$ the central velocity remains linear with Re to within 3 %. Exit arms in the present study were extended 9 diameters to minimize shape changes. (b) Time tracing of anemometer signals, showing the onset of strongly separated flow at the junction between curved pipe and bifurcation for inlet $\overline{W} = 58$ cm/s ($Re \approx 700$). Flow in the inner arm remains steady even at higher Re. Flow separation changes the shape of the flow–flow curve, rendering it inconsistent with the calibration curve.

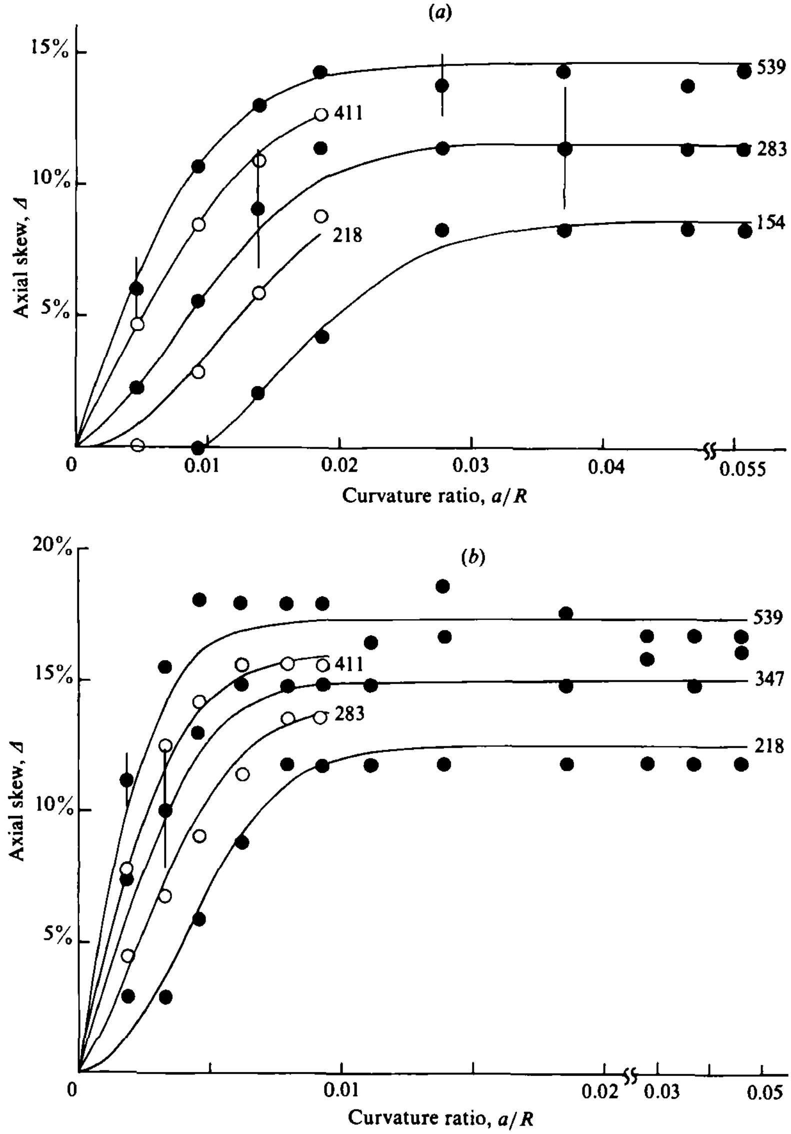

FIGURE 7. Measured skew Δ as a function of curvature ratio a/R. The solid curves represent (2) of text, with Re as parameter. (a) At an entrance length $L/a = 28.2$ the effect of curvature is apparent only in loosely coiled pipes ($a/R < 0.02$). (b) At $L/a = 56.5$ the regime for which curvature is important is even more restricted ($a/R \leqslant 0.007$).

where $\theta = L/R$ is expressed in radians. This expression also represents the downstream portion of the data given in figure 2.

Figure 7 (a, b) demonstrates how the developing axial profile can be characterized by two very different flow regimes: one where skew is a strong function of curvature and a second where skew is independent of R. In loosely coiled pipes where the total bend deflection θ is small, the outward shift in flow is a strong function of a/R, becoming nearly maximal for $\theta \sim 30°$. More tightly coiled pipes constitute an asymptotic regime, in which the degree of curvature has no bearing on the magnitude

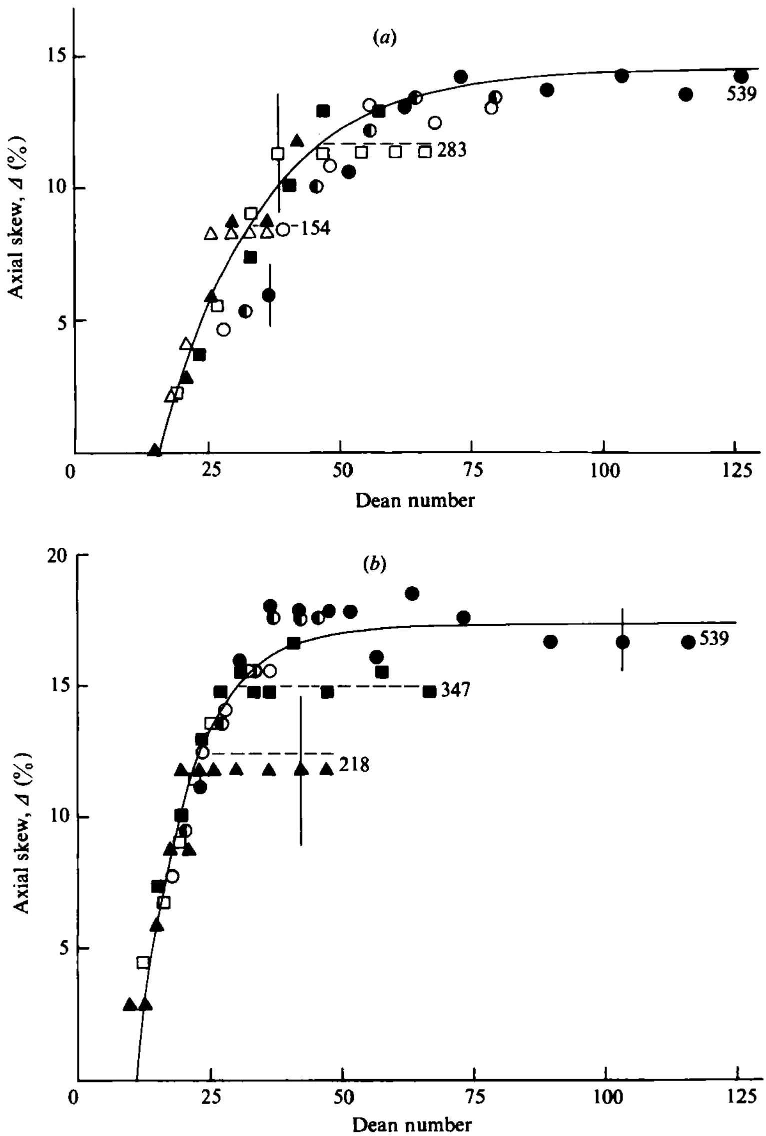

FIGURE 8. Data as given in figure 7, but rescaled to $\kappa = (a/R)^{\frac{1}{2}} Re$ to emphasize the onset of skew at finite Dean numbers ($11 \leqslant \kappa_c \leqslant 16$). (*a*) $L/a = 28.2$, $\kappa_c = 16$; (*b*) 56.5, 11. The solid curves forming the envelope of points are given by (3) of text; dashed lines are asymptotic values of skew with *Re* as parameter. △, $Re = 154$; ▲, 218; □, 283; ■, 347; ○, 411; ◐, 475; ●, 539.

of skew. The remainder of this section contains a more complete account of how axial skew develops in each of these two regimes.

In loosely coiled pipes, Δ increases with θ until ultimately attaining its asymptotic value Δ_0. A comparison of the curves in figures 7(*a*) and (*b*) reveals that the low-curvature regime becomes steadily more limited as L/a (and θ) increases. Thus, while stronger curvature appears to hasten the *development* of skew, the ultimate amplitude of skew for a given *Re* is limited by the plateau value Δ_0, independent of a/R. This conclusion can be made more quantitative by correlating Δ to the Dean

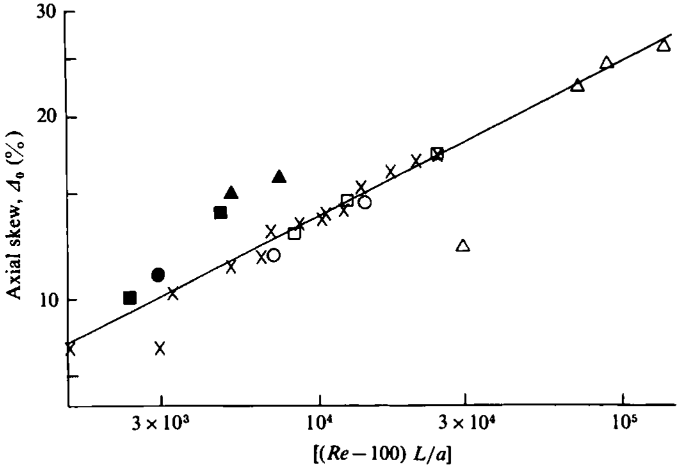

FIGURE 9. Log–log plot comparing the expression for asymptotic skew $\Delta_0 = 1.38[(Re-100)\,L/a]^{\frac{1}{4}}$ (solid line) with data for $L/a \geqslant 10$. From Agrawal *et al.*: ▲ $a/R = \frac{1}{7}$, $Re = 485$; △, $\frac{1}{20}$, 2527. From Olson & Snyder: ●, ○, $a/R = \frac{1}{4.66}$, $Re = 300$ and 1080. ■, □, $\frac{1}{8}$, 290 and 1100. Present report: ×.

number $\kappa = (a/R)^{\frac{1}{2}}Re$, as in figure 8(*a*, *b*). Data for loosely coiled pipes fall along an envelope of points that can be expressed empirically by

$$\Delta = 6.3\left(\frac{L}{a}\right)^{\frac{1}{4}}\left\{1-\exp\left[-\frac{\kappa-\kappa_c}{0.168L/a}\right]\right\} \quad (150 \leqslant Re \leqslant 540), \tag{3}$$

as depicted by the solid curves in this figure. For each *Re*, horizontal-plateau curves of asymptotic skew diverge from the envelope at specific values of a/R. This pattern is reminiscent of friction-factor curves (e.g. White 1929), except that in the latter case divergencies arise only at much higher values of *Re*. Like White's friction-factor data, figure 8(*a*, *b*) indicates that the onset of skew occurs at a small but finite Dean number, $11 \leqslant \kappa_c \leqslant 16$. This constitutes indirect evidence that a third flow regime may exist near zero Dean number, in which the axial-velocity profile remains axisymmetric throughout the pipe cross-section.

In the asymptotic regime of tightly coiled pipes, axial skew is independent of bend curvature even when flow is not yet fully developed. A quantitative comparison of figures 7(*a*) and (*b*) makes this evident, in that for each $Re \geqslant 200$ asymptotic skew Δ_0 is much more pronounced at $L/a = 56.5$. This result is consistent with earlier observations given in figure 2; at high *Re*, Δ_0 simply increases as $[Re\,L/a]^{\frac{1}{4}}$ in regions sufficiently far downstream to be decoupled from the inlet line vortex. However, at low *Re* axial skew seems to depart from a simple $\frac{1}{4}$-power relation. A better fit to all our data can be obtained by postulating that the onset of skew in the asymptotic regime may require a critical $Re_c = 100 \pm 50$. Unfortunately, the departure is significant only at low *Re*, for which the experimental uncertainty in our technique is too large to permit a direct resolution of this point. Qualitatively, however, the data for both entrance lengths exhibit such a trend. Using this model permits all the experimental data to be least-squares fitted to the expression

$$\Delta_0 = 1.38\left[(Re-100)\frac{L}{a}\right]^{\frac{1}{4}}. \tag{4}$$

This relation is compared in figure 9 with data obtained for curved pipe segments 1.91 and 3.81 cm i.d. with $L/a \geqslant 10$ and $150 \leqslant Re \leqslant 2600$.†

A complete expression for axial skew valid over the stated limits to our data is given by (2), which combines the effects of each regime. Thus we have demonstrated experimentally that axial skew may develop by either of two qualitatively different modes: as a function only of the parameter $[(Re-100)\,L/a]$ independent of curvature; or else, in loosely coiled pipes, as a function of κ and l/a. The demarcation between the two regimes depends strongly on a/R.

4. Discussion

An elementary analysis of the effect of stream curvature on flow stability (e.g. Landau & Lifshitz 1959) requires centrifugal force to be balanced by a radial pressure gradient $\partial p/\partial r = \rho W^2/R$, where W is the local axial velocity. Rayleigh's stability principle is then used to show that a stable flow configuration is achieved only if the axial-velocity profile is skewed towards the outside of the pipe bend. Thus, in the absence of other contributions the centreline-velocity gradient, $\partial W/\partial r \approx (\overline{W}/a)\Delta$ could be expected to vary inversely with stream curvature. Experimentally we find Δ_0 to be independent of R for tightly coiled pipes, so that this simple theoretical model is not accurate. We believe that the actual patterns of skew are influenced by strong secondary currents induced throughout the cross-section of a curved pipe. These currents produce a convective effect which tends to redistribute axial momentum, and thus opposes the action of centrifugal acceleration. The currents and the centrifugal acceleration both depend on R but they must somehow balance, even in developing flow, to make Δ_0 independent of R.

An analogous balance for *developed flow* was illustrated by Taylor (1929), who observed the characteristic helical motion of streamlines by injecting dye into the wall boundary layer of glass coils. Each dye streakline usually traced a semicircular helix whose cross-section was bounded by the centreline axis of symmetry. By measuring the helical pitch required to complete one circuit of the dye, he was able to estimate the strength of secondary circulation relative to the axial flow:

$$\text{Helical pitch} = \oint \left[\frac{\text{local axial velocity}}{\text{local transverse velocity}}\right] \mathrm{d}s.$$

He found pitch to be independent of primary axial velocity over a limited range of flow rates, and observed that for pipes of two different curvature ratios – 18.7 and 31.9 – the circuit was completed 'in about half a turn of the helix', so that pitch was approximately $[\pi/(2+\pi)]\,R/a$ in each case. Thus in developed flow the magnitude of transverse swirl increases directly with axial velocity.

Taylor's observations cannot be applied directly to our results because pitch is limited largely by the small transverse centreline currents, and therefore can be expected to be insensitive to axial skew. Nonetheless his description of helical motion exemplifies the invariance which can result from a balance of opposing flow mechanisms. We interpret the asymptotic regime of *developing* axial skew in a similar fashion: no effect of stream curvature is apparent because both transverse currents and centrifugal effects must remain balanced by the same dependence on curvature.

† From earlier results we surmise that axial-flow development is nearly inviscid for $L/a \lesssim 10$. Moreover, those data depart considerably from the expression for Δ_0.

By this view, axial skew is relatively underdeveloped in the loosely coiled regime. Thus we infer that in loosely coiled pipes the manifestation of centrifugal effects lags the growth of transverse secondary currents, and that the cumulative action of centrifugal acceleration can be promoted by increasing either the curvature ratio a/R or the entrance length L/a.

The existence of two dissimilar modes of axial-flow development is consonant with the results of a numerical investigation by van Dyke (1978), who used a series expansion to evaluate curved-pipe friction-factor ratios λ_c/λ_s. He found that λ_c/λ_s increased as $\kappa^{\frac{1}{4}}$, contrary to a square-root dependence predicted by all boundary-layer theories. Van Dyke conjectured that these two analytic methods could be reconciled if the asymptotic behaviour of secondary flow depended on the separate effects of a/R and Re, rather than on κ uniquely. Our experimental results given in figure 8 (a, b) show just such an effect: for a given Dean number $\kappa > 25$ axial skew can take on a multiplicity of values, with a/R or Re as an additional independent parameter.

Van Dyke's findings have been considered to be controversial because they seem to conflict with experimental results and with other numerical analyses. However, his results are in agreement with measured friction-factor ratios in the limit of very loose coiling ($a/R \rightarrow 0$). In our view, other differences stem from the basic governing equations and not from van Dyke's method.

To date most computational studies, including that of van Dyke, have used a set of governing equations in which terms involving a/R are neglected except to lowest order. Hence this formulation is strictly valid only for loosely coiled pipes, but in practice the condition of loose coiling has been expected to hold for $a/R < 0.2$. Our experimental results in figure 8(b) indicate that this upper bound cannot generally be correct, since asymptotic skew begins to diverge from the Dean-number envelope of points at values of a/R that are at least as small as $(\kappa/Re)^2 = (\frac{30}{347})^2 \simeq \frac{1}{140}$. At higher a/R axial skew can no longer be correlated uniquely with κ. Since the low-order approximation to the full Navier–Stokes equations predicts that *all* flow patterns should scale *only* with κ, we can conclude that neglected terms of order $(a/R)^1$ or higher must become significant for $a/R > \frac{1}{140}$. This argument may account for van Dyke's observation that experimental values of friction-factor ratios diverged from his series solutions for $a/R > \frac{1}{250}$. Based on our experimental results, we believe that the governing equations he used are inaccurate for values of a/R greater than this bound. Likewise, we attribute other small but observable differences between experiment (Olson & Snyder 1985) and computation (Dennis & Ng 1982) to the use of this same low-order approximation.

Recently Soh & Berger (1984) have numerically solved the full Navier–Stokes equations for laminar entry flow in curved pipes with arbitrary curvature ratio. Their formulation should make it possible to obtain a more exact criterion for loose coiling, by generating baseline cases of secondary-flow patterns to evaluate the accuracy of the low-order approximation.

The extensive range of curvatures used in the present study settles a scaling ambiguity that was left unresolved in an earlier report by Olson & Snyder (1985). In that study the range of a/R was too limited to determine whether upstream axial skew should be scaled to L/a or to $L/(aR)^{\frac{1}{2}}$. Our present data, for $L/a = 28.2$ with $a/R = \frac{1}{52}$ and $\frac{1}{36}$, yield values of skew which are less than half those predicted by an empirical curve of $\Delta_0(a/R)^{\frac{1}{2}}$ *vs* $L/(aR)^{\frac{1}{2}}$. When scaled to L/a, these same values fall along the low-Re curve in figure 2. Likewise, the centreline-velocity profile in figure 6, measured at 90° of bend, corresponds to a far-downstream condition $L/a = 56.5$. Although it is more sharply peaked than any of the profiles in figure 1, it best matches

$\theta = 180°$, corresponding to $L/a = 25.1$. Thus L/a is most likely to be the upstream lengthscale for axial skew in *tightly coiled* pipes, a conclusion that is reinforced by the form of the downstream lengthscale, $[(Re-100)\,L/a]$, for asymptotic skew Δ_0. It would be premature, however, to extend this conclusion to include *loosely coiled* pipes, for which no data is yet available regarding the upstream region of flow development.

The present data indicate that other aspects of entry flow in curved pipes merit systematic investigation. For example, the onset of skew at a finite Dean number κ_c suggests how the length ratio L_c/L_s for fully developed flow could approach the physically reasonable limit 1 as a/R tends to zero. An analogous effect may occur in the tightly coiled regime, for $Re \sim Re_c$.† In comparing figures 7(*a*) and (*b*) an additional question arises: as entrance length increases, does the steady encroachment of asymptotic skew culminate in the complete disappearance of the loosely coiled regime? Physically this would signify that in fully developed flow the action of centrifugal acceleration is matched by transverse swirl for all $\kappa \geqslant \kappa_c$. We feel this is a point best resolved by numerical analysts, as fully developed flow is a difficult concept to define experimentally.

5. Conclusions

This report demonstrates that the downstream development of axial skew in steady laminar curved-pipe flow cannot be characterized by a single dimensionless parameter. In loosely coiled pipes the growth of axial skew depends strongly on pipe curvature, through the Dean number, but in more tightly coiled pipes the precise curvature is immaterial. Instead, the magnitude of asymptotic skew Δ_0 varies as $[(Re-100)\,L/a]^{\frac{1}{4}}$, independent of a/R. The existence of two such dissimilar modes serves to substantiate experimentally van Dyke's conjecture that the development of secondary flow in curved pipes involves a separate consideration of a/R and Re, and not of Dean number uniquely. This result also shows that higher-order terms involving the curvature ratio begin to modify the distribution of flow for values of a/R at least as small as $\frac{1}{140}$.

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† It is noteworthy that White (1929), from his experimental study of fully developed curved-pipe friction-factor ratios, found $\lambda_c/\lambda_s = 1$ for $\kappa \leqslant 11.6$ and also for $Re \leqslant 80$.

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